MHD Oscillatory Flow past a Semi-Infinite Plate

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Theme

LIGHTHILL[1] studied effects of small amplitude oscillations and Lin[2] studied effects of finite amplitude oscillations on the boundary-layer flow. Lighthill's results were experimentally confirmed by Hill and Stenning.[3] They assumed the freestream to consist of a constant mean velocity over which is superimposed a purely time-dependent oscillatory flow. Kestin et al.[4] and Patel[5] studied the effects of oscillatory progressive wave type of freestream on the boundary-layer flow. In the energy equation, the viscous dissipation effects were neglected by Kestin et al. So taking into account viscous dissipative heat, Kestin's problem was solved again by Takhar and Soundalgekar.[6] We now consider the effects of a transverse magnetic field on the flow past a semi-infinite plate, taking into account the progressive wave type of disturbance in the freestream. By assuming the disturbance in the form $U_\infty(t)=U_0(1+\epsilon e^{i\omega t})$, a similar problem was solved by Evans.[7] Evans solved it by assuming a series in powers of two parameters $s(\omega x/u_0)$, the Strouhal number, and $\chi(\sigma B_y^2 x/\rho U_0=H_a^2//Re)$, the magnetic field parameter. We have solved the problem by assuming a series in powers of $\sqrt{s}$ only and $\chi>1$. The induced magnetic field is assumed to be negligible (Cowling[8]) which is true when magnetic Reynolds numer is small.

Contents

For a two-dimensional MHD flow past a semi-infinite plate, with x-axis in the direction of flow and y-axis normal to the plate, the governing equations, neglecting induced magnetic field, are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0^2}{\rho}(U-u) \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 -\frac{u}{C_p}\left[\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\frac{\sigma B_0^2}{\rho}U\right]+\frac{\sigma B_0^2}{\rho C_p}u^2 \tag{3}$$

with the freestream as

$$u(x,t)=U_0\{1+\lambda\cos\omega[(x/U_0)-t]\},\ \lambda<1 \tag{4}$$

Here all quantities have their usual meaning except U_0 which is constant mean velocity, ω the frequency and λ the amplitude of the oscillation. In Eqs. (3), the effects due to viscous dissipation and stress work are considered.

The boundary conditions are

$$u(o)=v(o)=0,\quad T(o)=T_w,$$
$$u(\infty)=U(x,t),\qquad v(\infty)=0,\qquad T(\infty)=T_\infty \tag{5}$$

To solve these equations, we follow Kestin et al. and derive the following equations in nondimensional form

$$f'''+\frac{1}{2}ff''+M(1-f')=0 \tag{6}$$

$$\theta_p''+\frac{P}{2}f\theta_p'=-PE_c[f''^2-Mf'+Mf'^2] \tag{7}$$

$$A_1'''+\frac{1}{2}(fA_1''+f''A_1)+M(1-A_1')=0 \tag{8}$$

$$2K_1''+P(fK_1'+A_1\theta_p')+2PE_c[2A_1''f''-M(A_1'+f')+2MA_1'f']=0 \tag{9}$$

$$4C_1'''+2(fC_1''+f''C_1)+A_1A_1''-4MC_1'=0 \tag{10}$$

$$4M_1''+P(2fM_1'+A_1K_1'+2P\theta_p'C_1)=-2PE_c[A_1''^2+4f''C_1''-M(2C_1'+A_1')+M(A_1'^2+4f'C_1')] \tag{11}$$

with boundary conditions

$$f=f'=A_1=A_1'=K_1=C_1=C_1'=M_1=0,\quad \theta_p=1\quad \text{at}\quad \eta=0 \tag{12a}$$

$$f'=1,\quad \theta_p=0,\quad A_1'=1,\quad K_1=C_1'=M_1=0\quad \text{as}\quad \eta\to\infty \tag{12b}$$

Here primes denote differentiation with respect to η, and $P(\mu C_p/k)$, $Ec(U_o^2/C_p(T_w-T_\infty))$, and $M[(\sigma B_o^2 x^2/\rho\nu)/(xU_o/\nu)=Ha^2/Re]$ are, respectively, the usual Prandtl number, Eckert number, and magnetic field parameter. A_1, C_1, K_1, M_1 are terms in the series for u_1, θ_1, etc. and $\theta=T-T_w/T_w-T_\infty$. These equations were solved numerically on a computer and the results are shown on Figs. 1-5 for $P=0.733$, $Ec=0.01$. Following Kestin et al. the time-averaged skin-friction and rate of heat transfer are derived as follows

$$\bar{C}_f=\frac{2f''(o)}{\sqrt{R}}[1+\lambda^2C_1''(o)/f''(o)]$$

$$\overline{Nu}=Nu_o[1+\lambda^2M_1'(o)/\theta_p'(o)]$$

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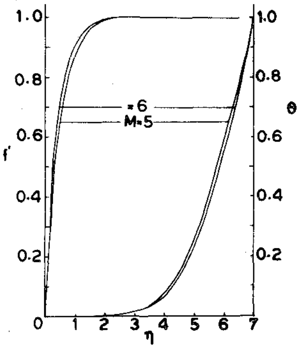


Fig. 1 Velocity and temperature profiles: $P=0.733$, $E=0.01$.

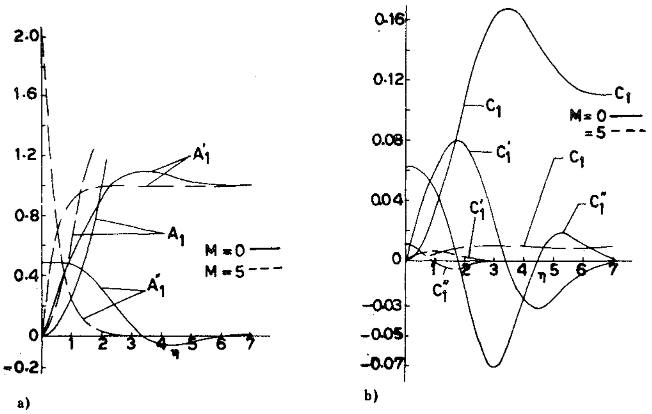
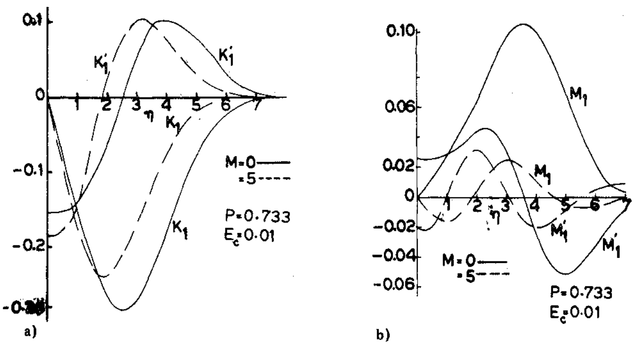


Fig. 2 The functions and their derivatives: a) A_I; b) C_I.



Fog. 3 The functions and their derivatives: a) K_I; b) M_I.

Table 1 Values

M	$C_I''(o)/f''(o)$	$M_I''(o)/\theta_o''(o)$
4	0.0072	0.013
5	0.0054	0.051
6	0.0043	0.081

The numerical values of $C_I''(o)/f''(o)y$ and $M_I''(o)/\theta_p'(o)$ are entered in Table 1. We conclude from this table that an increase in M leads to a decrease in the time-average skin-friction and an increase in the time-averaged rate of heat transfer, as compared to the nonmagnetic case. We now compare our results with those of Evans. For small M, the skin-friction and Nu increase with increasing M and the same is true in our case for $M>1$. The effects of Strouhal number have also been found similar to those of Evans. Hence we conclude that the effects of time-dependent disturbances or wave-type disturbances are the same on boundary-layer flows.

Acknowledgment

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